Phenomenological model of the Kaonic Nuclear Cluster $K^- pnn$ in the ground state

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Abstract

A phenomenological model, proposed for the Kaonic Nuclear Cluster (KNC) $K^- pp$ (or $\frac{2}{\bar{K}}\mathrm{H}$), is applied to the theoretical analysis of the KNC $K^- pnn$ (or S^0). The theoretical values of the binding energy and the width are equal to $\epsilon_{S^0} = -197\,\mathrm{MeV}$ and $\Gamma_{S^0} = 16\,\mathrm{MeV}$. They agree well with the experimental data $\epsilon_{S^0}^{\mathrm{exp}} = -194.0^{+1.5}_{-4.4}\,\mathrm{MeV}$ and $\Gamma_{S^0}^{\mathrm{exp}} < 21\,\mathrm{MeV}$ (PLB **597**, 263 (2004)) and the theoretical values $\epsilon_{S^0} = -191\,\mathrm{MeV}$ and $\Gamma_{S^0} = 13\,\mathrm{MeV}$, obtained by Akaishi and Yamazaki within the potential model approach.

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1 Introduction

Recently [1] Suzuki *et al.* have announced the experimental discovery of the bound state $K^- pnn$, named $S^0(3115)$, with quantum numbers $I(J^P) = 1(\frac{3}{2}^+)$, the binding energy $\epsilon_{S^0}^{\exp} = -194.0^{+1.5}_{-4.4}\,\mathrm{MeV}$ and the width constrained by $\Gamma_{S^0}^{\exp} < 21\,\mathrm{MeV}$. The possible existence of such a Kaonic Nuclear Cluster (KNC) as well as of the KNC $K^- pp$ (or $\frac{2}{K}\mathrm{H}$), which has been observed by the FINUDA Collaboration [2], has been pointed out by Akaishi and Yamazaki within the potential model approach [3]–[5]. According to [1], the main decay channels of the KNC $S^0(3115)$ are $S^0 \to \Sigma^- pn$, $S^0 \to \Sigma^- d$ and $S^0 \to \Sigma^0 nn$, where d is the deuteron. The decay channel $S^0 \to \Lambda^0 nn$ is suppressed.

In this paper we extend the phenomenological quantum field theoretic model for the KNC $\frac{2}{K}\mathrm{H}$, proposed in [6], to the description of the KNC $S^0(3115)$. The binding energy and the width of the KNC $S^0(3115)$ we define as [6]

$$ -\epsilon_{S^0} + i\,\frac{\Gamma_{S^0}}{2} = \int d\tau\, \Phi_{S^0}^*\, M(K^- pnn \to K^- pnn)\, \Phi_{S^0}, \qquad (1.1) $$

where $d\tau$ is an element of the phase space of the $K^- pnn$ system, Φ_{S^0} is the wave function of the KNC $S^0(3115)$ and $M(K^- pnn \to K^- pnn)$ is the amplitude of $K^- pnn$ scattering.

In our model for the calculation of the scattering amplitude of the constituents of the KNC we use the chiral Lagrangian with a non–linear realization of chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings [7], accepted for the analysis of strong low–energy interactions in ChPT [8] (see also [9, 10]). We calculate these amplitudes in the tree–approximation to leading order in the large N_C and chiral expansion, where N_C is the number of quark colour degrees of freedom [11]. According to Witten [11], the large N_C expansion is equivalent to the heavy–baryon approximation, applied in ChPT [8]–[10] to the analysis of low–energy meson–baryon interactions (see also [12, 13]).

In our model the main part of correlations between constituents of the KNC are described by the wave function of the KNC.

For the construction of the wave function Φ_{S^0} one has to specify the structure of the KNC $S^0(3115)$. The simplest structure of the bound $K^- pnn$ system with quantum numbers $I(J^P) = 1(\frac{3}{2}^+)$ and the decay mode $S^0 \to \Lambda^0 nn$ suppressed is $(K^- p)_{I=0}(nn)_{I=1}^{\ell=1}$, where the $K^- p$ pair is in the state with isospin $I = 0$, i.e. $(K^- p)_{I=0}$, and the nn pair is in the P–wave and spin–triplet state with isospin $I = 1$, i.e. $(nn)_{I=1}^{\ell=1}$.

The paper is organised as follows. In Section 2 we calculate the binding energy of the KNC $S^0(3115)$. We get $\epsilon_{S^0} = -197\,\mathrm{MeV}$. According to [6], the dominant contribution comes from the Weinberg–Tomozawa term for $K^- pnn$ scattering. In Section 3 we calculate the partial widths of the decay modes $S^0 \to \Sigma^- pn$, $S^0 \to \Sigma^- d$, $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ and the total width of the KNC $S^0(3115)$. We get $\Gamma_{S^0} = 16\,\mathrm{MeV}$. We show that the dominant decay mode is $S^0 \to \Sigma^- pn$ with the partial width $\Gamma(S^0 \to \Sigma^- pn) = 15\,\mathrm{MeV}$. We calculate the invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay. In the Conclusion we discuss the obtained results. In the Appendix we define the wave function Φ_{S^0} of the KNC $S^0(3115)$.

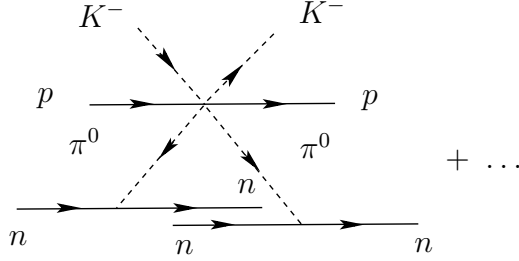Figure 1: Feynman diagrams giving in the tree–approximation contributions to the amplitude of the reaction $K^- pnn \to K^- pnn$ to next–to–leading order in the large N_C and chiral expansion with respect to the Weinberg–Tomozawa term.

2 Binding energy of the KNC $S^0(3115)$

According to [6], the binding energy of the KNC $S^0(3115)$ is given by

$$- \epsilon_{S^0} = \frac{5}{4} \frac{1}{F_\pi^2} \left(\frac{\mu_1^3 \omega_1^2 \Omega_1}{\pi^3} \right)^{1/2} = 197 \, \text{MeV}, \tag{2.1}$$

where $F_\pi = 92.4 \, \text{MeV}$ is the PCAC constant of pseudoscalar mesons. Other parameters μ_1, ω_1 and Ω_1 are related to the wave function Φ_{S^0} of the KNC $S^0(3115)$ and defined in the Appendix. The theoretical value agrees well with the experimental one $\epsilon_{S^0}^{\text{exp}} = -194.0^{+1.5}_{-4.4} \, \text{MeV}$. The binding energy is defined by the contribution of the Weinberg–Tomozawa term. This term gives the dominant contribution to the real part of the amplitude of elastic $K^- pnn$ scattering to leading order in the large N_C and chiral expansion.

The correction of next–to–leading order in the large N_C and chiral expansion to the Weinberg–Tomozawa term is defined by the diagrams in Fig. 1. The calculation of diagrams in Fig. 1 is carried out using the chiral Lagrangian with a non–linear realization of chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings [6]. For $N_C = 3$ such a correction makes up a few percent of the Weinberg–Tomozawa term.

3 Width of the KNC $S^0(3115)$

In this section we calculate the partial widths of the decay modes $S^0 \to \Sigma^- pn$, $S^0 \to \Sigma^- d$, $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ and the total width Γ_{S^0} of the KNC $S^0(3115)$. For the total width we get the value $\Gamma_{S^0} = 16 \, \text{MeV}$, agreeing well with the experimental data [1] $\Gamma_{S^0} < 21 \, \text{MeV}$ and the potential model prediction $\Gamma_{S^0} = 13 \, \text{MeV}$ [4] (see also [5]).

3.1 The $S^0 \to \Sigma^- pn$ decay mode

The Feynman diagrams of the reaction $K^- pnn \to \Sigma^- pn$ are depicted in Fig. 2. In the tree–approximation these diagrams give the main contribution to the amplitude of the $S^0 \to \Sigma^- pn$ decay to leading order in the large N_C and chiral expansion. The calculation of these diagrams is carried out using the chiral Lagrangian with chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings (see Appendix B of Ref.[6]). The partial
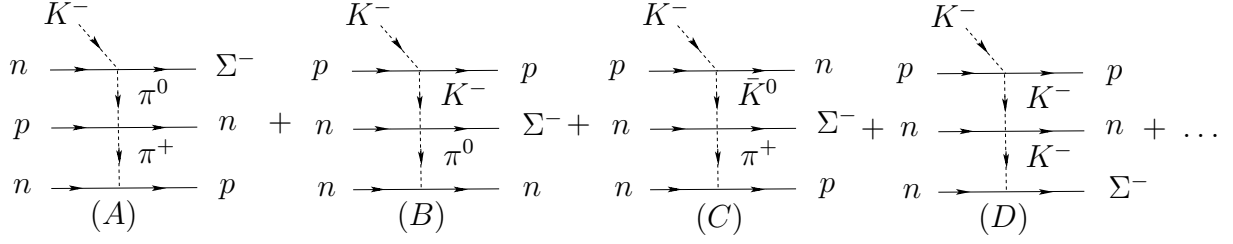
Figure 2: Feynman diagrams of the amplitude of the reaction $K^- pnn \to \Sigma^- pn$ giving in the tree–approximation the main contribution to leading order in the large N_C and chiral expansion.

width of the $S^0 \to \Sigma^- pn$ decay is equal to

$$
\begin{aligned}
\Gamma(S^0 \to \Sigma^- pn) &= \left(\frac{\mu_1^3 \omega_1^2 \Omega_1}{\pi^3}\right)^{1/2} \left(\frac{\mu_2 \omega_2}{\pi}\right)^{3/2} \left(\frac{\mu_3 \omega_3}{\pi}\right)^{5/2} \frac{9}{32\pi^2} \frac{g_{\pi NN}^2}{m_K m_\pi^4 F_\pi^8} \sqrt{\frac{2\mu^3}{m_N}} \\
&\times \varepsilon^2 \, f(\varepsilon) = 15 \, \text{MeV},
\end{aligned}
\tag{3.1}
$$

where $g_{\pi NN} = g_A m_N / F_\pi = 13.3$ is the πNN coupling constant [14, 15], $\varepsilon = m_{S^0} - 2m_N - m_\Sigma = 44 \, \text{MeV}$ is the energy excess calculated for $m_N = 940$ and $m_\Sigma = 1193 \, \text{MeV}$, $\mu = 2m_N m_\Sigma / (2m_N + m_\Sigma) = 730 \, \text{MeV}$ and $f(\varepsilon) = 0.78$ is the contribution of the phase volume of the $\Sigma^- pn$ state. The parameters μ_i, ω_i for (i=1,2,3) and Ω_1 of the wave function Φ_{S^0} of the KNC $S^0(3115)$ are defined in the Appendix.

3.2 Invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay

Following [16] we calculate the invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay. For this aim we use the following variables [16]

$$
X = m_{np}^2 = (E_n + E_p)^2 - (\vec{k}_n + \vec{k}_p)^2 \,, \ \ Y = m_{\Sigma p}^2 = (E_{\Sigma^-} + E_p)^2 - (\vec{k}_\Sigma + \vec{k}_p)^2,
\tag{3.2}
$$

which have the meaning of the invariant–squared masses of the np and $\Sigma^- p$ pairs, respectively. At the rest frame of the KNC $S^0(3115)$ in terms of the variables (X, Y) the kinetic energies of the neutron and the proton are given by

$$
T_n = \frac{(m_{S^0} - m_N)^2 - Y}{2m_{S^0}} \,, \ T_p = \varepsilon - \frac{(m_{S^0} - m_\Sigma)^2 + (m_{S^0} - m_N)^2 - X - Y}{2m_{S^0}}.
\tag{3.3}
$$

The invariant–mass spectrum or the Dalitz density distribution of the $S^0 \to \Sigma^- pn$ decay is defined by

$$
\begin{aligned}
m_{S^0}^3 \frac{d^2\Gamma(S^0 \to \Sigma^- pn)}{dX \, dY} &= \left(\frac{\mu_1^3 \omega_1^2 \Omega_1}{\pi^3}\right)^{1/2} \left(\frac{\mu_2 \omega_2}{\pi}\right)^{3/2} \left(\frac{\mu_3 \omega_3}{\pi}\right)^{5/2} \frac{9}{128\pi^2} \frac{g_{\pi NN}^2 m_\Sigma m_{S^0}}{m_K m_\pi^4 F_\pi^8} \\
&\times [A(X,Y) + B(X,Y) + C(X,Y) + D(X,Y)]^2 = \\
&= 10 \, [A(X,Y) + B(X,Y) + C(X,Y) + D(X,Y)]^2.
\end{aligned}
\tag{3.4}
$$

In the (X, Y)–plane the Dalitz domain is bounded by the curve with $X_{\min} = 4m_N^2$, $X_{\max} = (m_{S^0} - m_\Sigma)^2$, $Y_{\min} = (m_\Sigma + m_N)^2$ and $Y_{\max} = (m_{S^0} - m_N)^2$ [16]. The invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ is represented in Fig. 3.
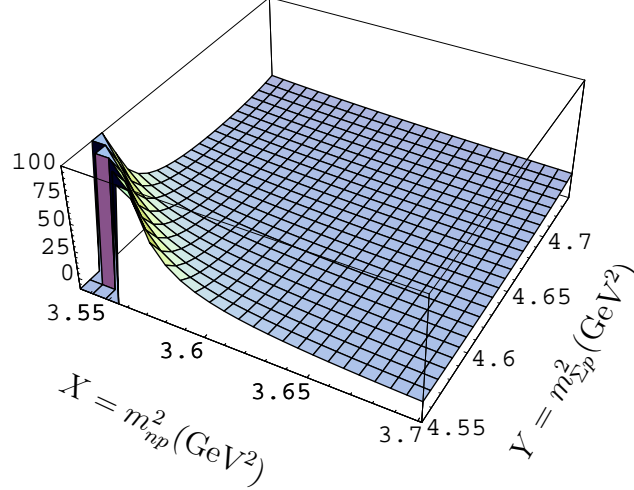
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Figure 3: The invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay in the variables $X = m_{np}^2$ and $Y = m_{\Sigma p}^2$ [16].

3.3   The $S^0 \to \Sigma^- d$ decay mode

The partial width of the $S^0 \to \Sigma^- d$ decay is equal to [17]

$$\Gamma(S^0 \to \Sigma^- d) = \left(\frac{\mu_1^3 \omega_1^2 \Omega_1}{\pi^3}\right)^{1/2} \left(\frac{\mu_2 \omega_2}{\pi}\right)^{3/2} \left(\frac{\mu_3 \omega_3}{\pi}\right)^{5/2} \frac{9\, g_{\pi NN}^2 m_\Sigma Q_{\Sigma d}}{4 m_K m_N m_{S^0} m_\pi^4 F_\pi^8}$$

$$\times \left| \int \frac{d^3 k}{(2\pi)^3} \frac{\Phi_d(k)}{1 - \frac{1}{2} r_{np}^t a_{np}^t k^2 - i\, a_{np}^t k}(A + B + C + D) \right|^2 = 1\,\mathrm{MeV}, \qquad (3.5)$$

where the amplitudes A, B, C and D are defined by the corresponding Feynman diagrams in Fig. 2, $Q_{\Sigma d} = 266\,\mathrm{MeV}$ is a relative momentum of the $\Sigma^- d$ pair, $\Phi_d(k) = \sqrt{8\pi\gamma}/(\gamma^2 + k^2)$ is the wave function of the deuteron in the ground state in the momentum representation and $\gamma = 1/R_d = 46\,\mathrm{MeV}$ [18]. The amplitude of the np scattering is defined according to [19] (see also [17]) and taken in the effective range approximation

$$k \cot \delta_{np}(k) = -\frac{1}{a_{np}^t} + \frac{1}{2} r_{np}^t k, \qquad (3.6)$$

where $\delta_{np}(k)$ is the phase shift of the np scattering in the $^3\mathrm{S}_1$ state, $a_{np}^t = (5.424 \pm 0.004)\,\mathrm{fm}$ and $r_{np}^t = (1.759 \pm 0.005)\,\mathrm{fm}$ are the S–wave scattering length and effective range of np scattering in the $^3\mathrm{S}_1$ state [18].

The main contribution to the integral comes from the relative momenta $k \sim 100\,\mathrm{MeV}$, and 94 % of the value of the integral are concentrated in the region $0 \le k \le 200\,\mathrm{MeV}$. For the relative momenta $k = 100\,\mathrm{MeV}$ and $k = 200\,\mathrm{MeV}$ the phase shift of the np scattering, defined by the effective range approximation (3.6), is equal to $\delta_{np} = 85.3^0$ and $\delta_{np} = 54.6^0$, respectively. These values agree well with the SAID analysis of the experimental data on the np scattering in the $^3\mathrm{S}_1$ state [20]: $\delta_{np} = 85.84^0$ and $\delta_{np} = 48.84^0$, respectively.
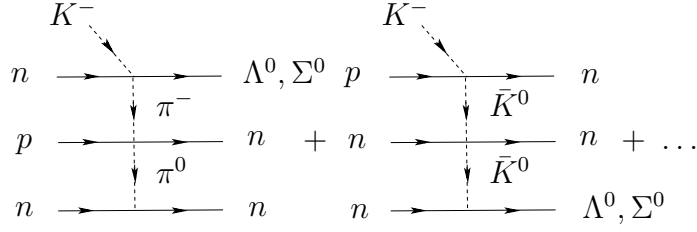
Figure 4: Feynman diagrams of the amplitudes of the reactions $K^- pnn \to \Lambda^0 nn$ and $K^- pnn \to \Sigma^0 nn$ giving in the tree–approximation the main contribution to leading order in the large N_C and chiral expansion.

3.4   The $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decay modes

The amplitudes of the $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decays are defined by the integrals of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$ with the wave function Φ_{S^0} of the KNC $S^0(3115)$. Feynman diagrams of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$ are depicted in Fig.4. In the tree–approximation these diagrams give the main contribution to leading order in the large N_C and chiral expansion. The contributions of the amplitudes of the reactions $K^- pnn \to \Sigma^0 nn$ and $K^- pnn \to \Lambda^0 nn$, calculated to leading order in the large N_C and chiral expansion, to the amplitudes of the $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ decays vanish. This means that the decay modes $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ are suppressed to leading order in the large N_C and chiral expansion.

3.5   Width of the KNC $S^0(3115)$

The width of the KNC $S^0(3115)$ is defined by the sum of the partial widths of all modes. Since the dominant modes are $S^0 \to \Sigma^- pn$ and $S^0 \to \Sigma^- d$, the width of the $S^0(3115)$ is equal to

$$\Gamma_{S^0} = 16 \, \text{MeV}. \qquad (3.7)$$

This agrees well with the experimental constraint $\Gamma_{S^0}^{\text{exp}} < 21 \, \text{MeV}$ and the theoretical value $\Gamma_{S^0} = 13 \, \text{MeV}$, predicted by Akaishi and Yamazaki within the potential model approach [4] (see also [5]).

4   Conclusion

The phenomenological quantum field theoretic model of the KNC $K^- pp$ (or $_{\bar{K}}^2 \text{H}$), proposed in [6], is extended to the description of the KNC $S^0(3115)$ with quantum numbers $I(J^P) = 1(\frac{3}{2}^+)$ [1]. The wave function of the KNC $S^0(3115)$ is taken in the oscillator form. The frequencies of oscillations are related to the frequency ω, describing longitudinal oscillations of the K^-–meson with respect to the pp pair in the KNC $_{\bar{K}}^2 \text{H}$ [6]. The value of the frequency $\omega = 64 \, \text{MeV}$ has been calculated in [6] by fitting the width of the $\Lambda^0(1405)$ hyperon, treated as a bound $K^- p$ state $_{\bar{K}}^1 \text{H}$.

We would like to make clear the title of our model namely "The phenomenological quantum field theoretic model". On the one hand our model is "phenomenological", since

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we construct the wave function of the KNC, the parameters of which are fixed in terms of the experimental data on the width and mass of the $\Lambda(1405)$ hyperon. On the other hand our model is "quantum field theoretic", since to the calculation of scattering amplitudes of the constituents of the KNC we apply the chiral Lagrangian with chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings, ChPT and the large N_C expansion.

The binding energy $\epsilon_{S^0} = -197.0 \, \text{MeV}$ and the width $\Gamma_{S^0} = 16 \, \text{MeV}$, calculated in our model, agree well with the experimental data $\epsilon_{S^0}^{\exp} = -194.0^{+1.5}_{-4.4} \, \text{MeV}$ and $\Gamma_{S^0}^{\exp} < 21 \, \text{MeV}$. We have found that the decay mode $S^0 \to \Sigma^- pn$ is dominant, whereas the decay modes $S^0 \to \Sigma^0 nn$ and $S^0 \to \Lambda^0 nn$ are suppressed. The calculation of the amplitudes of elastic and inelastic $K^- pnn$ scattering we have carried out using the chiral Lagrangian with the non–linear realization of chiral $SU(3) \times SU(3)$ symmetry and derivative meson–baryon couplings (see Appendix B of Ref.[6]). We have kept the contributions of leading order in the large N_C and chiral expansion [6]. In more detail the experimental analysis of the theoretical amplitude of the $S^0 \to \Sigma^- pn$ decay can be obtained through the measurements of the invariant–mass spectrum [16]. The theoretical invariant–mass spectrum of the $S^0 \to \Sigma^- pn$ decay is given by Eq.(3.4) and represented in Fig. 3.

We would like to accentuate that our theoretical predictions for the binding energy $\epsilon_{S^0} = -197.0 \, \text{MeV}$ and the width $\Gamma_{S^0} = 16 \, \text{MeV}$ of the KNC $S^0(3115)$ agree well with results, obtained by Akaishi and Yamazaki within the potential model approach [4] (see also [5]): $\epsilon_{S^0} = -191.0 \, \text{MeV}$ and $\Gamma(S^0 \to \Sigma^- pn) = 13 \, \text{MeV}$.

However one can show that in our model the nuclear matter density of the KNC $S^0(3115)$ is smaller by factor 3 than $n_{S^0}(0) = 1.56 \, \text{fm}^{-3}$, predicted in the potential model approach [5]. Indeed, using the wave function of the KNC $S^0(3115)$ one can calculate the nuclear matter density at the center of mass of the $K^- pnn$ system. This gives

$$n_{S^0}(0) = 3 \left(\frac{\mu_1^3 \omega_1^2 \Omega_1}{\pi^3} \right)^{1/2} = 0.53 \, \text{fm}^{-3}. \tag{4.8}$$

The obtained result is larger by a factor 4 than the normal nuclear matter density $n_0 = 0.14 \, \text{fm}^{-3}$, but it is smaller by factor 3 than $n_{S^0}(0) = 1.56 \, \text{fm}^{-3}$, predicted by Akaishi and Yamazaki.

Our approach for the analysis of the Kaonic Nuclear Clusters $K^- p$ (or $\Lambda^0(1405)$), $K^- pp$ and $K^- pnn$ can be applied to the description of the Kaonic Nuclear Clusters $K^- ppn$ with isospin $I = 0$, $K^- ppn$ and $K^- ppp$ with isospin $I = 1$, $K^- NNNN$ and so on [5].

Appendix A: Wave function Φ_{S^0} of the KNC $S^0(3115)$

In the momentum representation the wave function Φ_{S^0} of the KNC $S^0(3115)$ is defined by [6]

$$\Phi_{S^0}(\vec{q}, \vec{k}, \vec{Q})_{JM} = \Phi_K(\vec{q}) \Phi_{p(nn)}(\vec{k}) \Phi_{nn}(Q) \Psi_{JM}(\vec{Q}). \tag{A-1}$$

This wave function describes the $K^- pnn$ system with total momentum $J = \frac{3}{2}$ and positive parity, where $\Phi_K(\vec{q})$, $\Phi_{p(nn)}(\vec{k})$ and $\Phi_{nn}(\vec{Q})$ are the wave functions of harmonic oscillators [6]

$$\Phi_K(\vec{q}) \;=\; \left(\frac{64\pi^3}{\mu_1^3 \omega_1^2 \Omega_1} \right)^{1/4} \exp\left(-\frac{\vec{q}_\perp^2}{2\mu_1\omega_1} - \frac{\vec{q}_\parallel^2}{2\mu_1\Omega_1} \right),$$

$$\Phi_{p(nn)}(\vec{k}) = \left(\frac{4\pi}{\mu_2\omega_2}\right)^{3/4} \exp\left(-\frac{\vec{k}^2}{2\mu_2\omega_2}\right),$$

$$\Phi_{nn}(\vec{Q}) = \Phi_{nn}(Q)\, Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}}) =$$

$$= \frac{8}{\sqrt{3}}\left(\frac{\pi}{\mu_3\omega_3}\right)^{5/4} |\vec{Q}| \exp\left(-\frac{\vec{Q}^2}{2\mu_3\omega_3}\right) Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}}), \qquad \text{(A-2)}$$

where the nn pair is in the P–wave state, $Y_{1m}(\vartheta_{\vec{Q}}, \varphi_{\vec{Q}})$ is a spherical harmonic, $m = 0, \pm 1$ is a magnetic quantum number. The frequencies and the reduced masses are

$$\omega_1 = \sqrt{\frac{3}{2}\frac{\mu}{\mu_1}}\,\omega = 76\,\text{MeV}\,,\ \mu_1 = \frac{3m_K m_N}{m_K + 3m_N} = 420\,\text{MeV}$$

$$\Omega_1 = \sqrt{3}\,\omega_1 = 132\,\text{MeV},$$

$$\omega_2 = \sqrt{\frac{\mu}{\mu_2}}\,\omega = 51\,\text{MeV}\,,\ \mu_2 = \frac{2}{3}\,m_N = 630\,\text{MeV},$$

$$\omega_3 = \frac{1}{2}\sqrt{\frac{\mu}{\mu_3}}\,\omega = 30\,\text{MeV}\,,\ \mu_3 = \frac{1}{2}\,m_N = 470\,\text{MeV}, \qquad \text{(A-3)}$$

where $\omega = 64\,\text{MeV}$ is the frequency of the longitudinal oscillation of the K^-–meson relative to the pp pair in the KNC $^2_{\bar{K}}\text{H}$ [6] and $\mu = 2m_K m_N/(m_K + 2m_N) = 391\,\text{MeV}$ is the reduced mass of the $K^-(pp)$ system, calculated for $m_K = 494\,\text{MeV}$ and $m_N = 940\,\text{MeV}$. The value $\omega = 64\,\text{MeV}$ is obtained from the fit of the width of the $\Lambda^0(1405)$ hyperon, treated as a bound K^-p state [6].

The wave functions $\Phi_K(\vec{q})$, $\Phi_{p(nn)}(\vec{k})$ and $\Phi_{nn}(\vec{Q})$ are normalised by

$$\int \frac{d^3q}{(2\pi)^3}|\Phi_K(\vec{q})|^2 = \int \frac{d^3k}{(2\pi)^3}|\Phi_{p(nn)}(\vec{k})|^2 = \int \frac{d^3Q}{(2\pi)^3}|\Phi_{nn}(\vec{Q})|^2 = 1. \qquad \text{(A-4)}$$

This gives

$$\int \frac{d^3q}{(2\pi)^3}\Phi_K(\vec{q}) = \left(\frac{\mu_1^3\omega_1^2\Omega_1}{\pi^3}\right)^{1/4}, \int \frac{d^3k}{(2\pi)^3}\Phi_{p(nn)}(\vec{k}) = \left(\frac{\mu_2\omega_2}{\pi}\right)^{3/4},$$

$$\int \frac{d^3Q}{(2\pi)^3}\,Q\,\Phi_{nn}(Q) = \frac{12\pi}{\sqrt{6}}\left(\frac{\mu_3\omega_3}{\pi}\right)^{5/4}. \qquad \text{(A-5)}$$

The wave functions $\Phi_K(\vec{q})$, $\Phi_{p(nn)}(\vec{k})$ and $\Phi_{nn}(\vec{Q})$ describe oscillations of the K^-–meson relative to the pnn system, the proton relative to the nn pair and the neutrons in the nn pair, respectively. The K^-pnn system with the wave function (A-1) possesses positive parity.

Since the total angular momentum of the KNC $S^0(3115)$ is $J = \frac{3}{2}$ [1, 4, 5], the wave functions $\Psi_{JM}(\vec{Q})$ of the angular momentum of the KNC $S^0(3115)$ are defined by

$$\Psi_{\frac{3}{2},+\frac{3}{2}} = \frac{1}{2}Y_{1,+1}\left[\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} + \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},+\frac{1}{2}}\right]\chi^{(p)}_{\frac{1}{2},+\frac{1}{2}} - \frac{1}{\sqrt{2}}Y_{1,0}\,\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(p)}_{\frac{1}{2},+\frac{1}{2}}$$

$$\Psi_{\frac{3}{2},+\frac{1}{2}} = \frac{1}{\sqrt{12}}Y_{1,+1}\left[\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} + \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},+\frac{1}{2}}\right]\chi^{(p)}_{\frac{1}{2},-\frac{1}{2}} + \frac{1}{\sqrt{3}}Y_{1,+1}\,\chi^{(1)}_{\frac{1}{2},-\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},-\frac{1}{2}}\chi^{(p)}_{\frac{1}{2},+\frac{1}{2}}$$

$$- \frac{1}{\sqrt{6}}Y_{1,0}\,\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(p)}_{\frac{1}{2},-\frac{1}{2}} - \frac{1}{\sqrt{3}}Y_{1,-1}\,\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(2)}_{\frac{1}{2},+\frac{1}{2}}\chi^{(p)}_{\frac{1}{2},+\frac{1}{2}},$$

$$\Psi_{\frac{3}{2},-\frac{1}{2}} = \frac{1}{\sqrt{3}} Y_{1,+1} \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(p)}_{\frac{1}{2},-\frac{1}{2}} + \frac{1}{\sqrt{6}} Y_{1,0} \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(p)}_{\frac{1}{2},+\frac{1}{2}}$$

$$- \frac{1}{\sqrt{3}} Y_{1,-1} \chi^{(1)}_{\frac{1}{2},+\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},+\frac{1}{2}} \chi^{(p)}_{\frac{1}{2},-\frac{1}{2}} - \frac{1}{\sqrt{12}} Y_{1,-1} \left[\chi^{(1)}_{\frac{1}{2},+\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} + \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},+\frac{1}{2}} \right] \chi^{(p)}_{\frac{1}{2},+\frac{1}{2}},$$

$$\Psi_{\frac{3}{2},-\frac{3}{2}} = \frac{1}{\sqrt{2}} Y_{1,0} \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(p)}_{\frac{1}{2},-\frac{1}{2}} - \frac{1}{2} Y_{1,-1} + \chi^{(1)}_{\frac{1}{2},-\frac{1}{2}} \chi^{(2)}_{\frac{1}{2},+\frac{1}{2}} \right] \chi^{(p)}_{\frac{1}{2},+\frac{1}{2}}, \tag{A-6}$$

where $\chi^{(a)}_{\frac{1}{2},\pm\frac{1}{2}}$ with $a = 1, 2, p$ are the spinorial wave functions of the neutrons and the proton, respectively.

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